Today's Plan:

Learning Target (standard): I will review for the semester exam.

Students will: Complete practice problems over previous concepts at the boards and study for my exam.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of exam problems.

Assessment: Board work

Differentiation: Students will work at the board, actively engage in practice review concepts with the aid of other students and the teacher.

Applied Calculus II Name______________________________

Final Exam Review

$y = mx + b$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

1) $f(x) = -(-2x+2)^{\frac{1}{2}}$ at $\boxed{(-1, -2)}$

$f'(x) = -\frac{1}{2}(-2x+2)^{-\frac{1}{2}}(-2)$

$f'(x) = (-2x+2)^{-\frac{1}{2}}$

$m_{tangent}$

$f'(-1) = (2+2)^{-\frac{1}{2}}$

$f'(-1) = 4^{-\frac{1}{2}} = \frac{1}{2} = m$

$y = mx+b$

$-2 = \frac{1}{2}(-1) + b$

$-2 = -\frac{1}{2} + b$

$b = -\frac{3}{2}$

$\boxed{y = \frac{1}{2}x - \frac{3}{2}}$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

normal $\perp$ tangent

2) $f(x) = x^3 - 10x^2 + 33x - 40$ at $(2, -6)$

$f'(x) = 3x^2 - 20x + 33$

$f'(2) = 3(2)^2 - 20(2) + 33$

$= 12 - 40 + 33$

$f'(2) = 5$ – $m_{tangent}$

$m_{normal} = -\frac{1}{5}$

$y = mx+b$

$-6 = -\frac{1}{5}(2) + b$

$-6 = -\frac{2}{5} + b$

$b = \frac{-28}{5}$

$\boxed{y = -\frac{1}{5}x - \frac{28}{5}}$

Solve each optimization problem.

3) A company has started selling a new type of smartphone at the price of $120 - 0.05x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$40 and the labor and overhead for running the plant cost \$6000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

Income $= (120 - .05x)x$

Cost $= 40x + 6000$

Profit = Income - Cost

$P(x) = (120x - 0.05x^2) - (40x + 6000)$

$P(x) = -.05x^2 + 80x - 6000$

$P'(x) = -.1x + 80$

$0 = -.1x + 80$

$.1x = 80$

$x = 800$

$P''(x) = -.1$

$P''(800) = -.1$

$P''(800) < 0$

max

$\therefore$ sell 800 smartphones

4) Which points on the graph of $y = 5 - x^2$ are closest to the point $(0, 2)$?

$\left(-\frac{\sqrt{10}}{2}, \frac{5}{2}\right), \left(\frac{\sqrt{10}}{2}, \frac{5}{2}\right)$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(x-0)^2+(5-x^2-2)^2}$

$d=\sqrt{x^2+(3-x^2)^2}$

$d=\sqrt{x^2+9-6x^2+x^4}$

$d=\sqrt{x^4-5x^2+9}$

$d(x)=x^4-5x^2+9$

$d'(x)=4x^3-10x$

$0=2x(2x^2-5)$

Critical #s:

$x=\cancel{0}, -\frac{\sqrt{10}}{2}, \frac{\sqrt{10}}{2}$

$y=5-\left(\frac{\sqrt{10}}{2}\right)^2$

$y=5-\frac{10}{4}$

$y=5-\frac{5}{2}$

$y=\frac{5}{2}$

$d''(x)=12x^2-10$

$d''\left(-\frac{\sqrt{10}}{2}\right)=d''\left(\frac{\sqrt{10}}{2}\right)=30-10>0$

min

Use the 1st Derivative Test to describe the function.

5) $y=\frac{x^4}{4}-\frac{5x^2}{4}$

D: $\mathbb{R}$

$y=\frac{1}{4}x^4-\frac{5}{4}x^2$

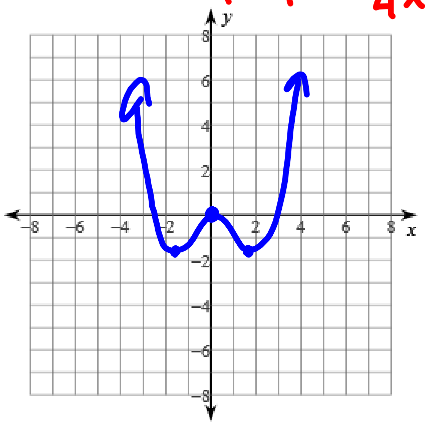
$y'=x^3-\frac{5}{2}x$

$0=x\left(x^2-\frac{5}{2}\right)$

$0=x\left(x+\frac{\sqrt{10}}{2}\right)\left(x-\frac{\sqrt{10}}{2}\right)$

Critical #s:

$x=-\frac{\sqrt{10}}{2}, 0, \frac{\sqrt{10}}{2}$



Domain Interval	x	$x+\frac{\sqrt{10}}{2}$	$x-\frac{\sqrt{10}}{2}$	$f'(x)$	$f(x)$
$\left(-\infty, -\frac{\sqrt{10}}{2}\right)$	−	−	−	−	decreasing
					> min $=\frac{25}{14}$ @ $x=-\frac{\sqrt{10}}{2}$
$\left(-\frac{\sqrt{10}}{2}, 0\right)$	−	+	−	+	increasing
					> max $=0$ @ $x=0$
$\left(0, \frac{\sqrt{10}}{2}\right)$	+	+	−	−	decreasing
					> min $=-\frac{25}{16}$ @ $x=\frac{\sqrt{10}}{2}$
$\left(\frac{\sqrt{10}}{2}, \infty\right)$	+	+	+	+	increasing