Today's Plan:

Learning Target (standard): I will review for the semester exam.

Students will: Complete practice problems over previous concepts at the boards and study for my exam.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of exam problems.

Assessment: Board work

Differentiation: Students will work at the board, actively engage in practice review concepts with the aid of other students and the teacher.

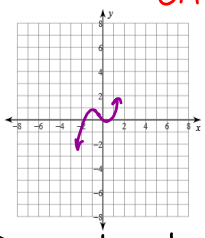
Use the 2nd Deriviative Test to describe the function.

6) $y = \frac{x^3}{6} + \frac{x^2}{6} - \frac{x}{6}$

$y = \frac{1}{6}x^3 + \frac{1}{6}x^2 - \frac{1}{6}x$ $\quad y' = \frac{1}{2}x^2 + \frac{1}{3}x - \frac{1}{6}$

D: $\mathbb{R}$ $\quad y' = \frac{1}{6}(3x^2 + 2x - 1)$

$y' = \frac{1}{6}(3x - 1)(x + 1)$



critical #s: $x = -1, \frac{1}{3}$

$y'' = x + \frac{1}{3}$ $\quad x = -\frac{1}{3}$ $\quad$ D: $\mathbb{R}$

$0 = x + \frac{1}{3}$ $\quad$ Possible POIs: $x = -\frac{1}{3}$

Domain Interval	$x + \frac{1}{3}$	$f''(x)$	$f(x)$
$(-\infty, -\frac{1}{3})$	$-$	$-$	concave down
$(-\frac{1}{3}, \infty)$	$+$	$+$	concave up

POI: $(-\frac{1}{3}, \frac{11}{162})$

Extremum:

$f''(-1) < 0$ max $= \frac{1}{6}$ @ $x = -1$

$f''(\frac{1}{3}) > 0$ min $= -\frac{3}{162}$ @ $x = \frac{1}{3}$

Describe the motion of the particle.

7) $s(t) = -t^3 + 23t^2 - 120t$ $\quad t > 0$

$a(t) = v'(t) = s''(t)$

$v(t) = s'(t)$

$v(t) = -3t^2 + 46t - 120$

$a(t) = -6t + 46$

$0 = -6t + 46$

$6t = 46$

$t = \frac{23}{3}$

$0 = -1(3t^2 - 46t + 120)$

$0 = -1(3t - 10)(t - 12)$

$t = \frac{10}{3}, 12$

∴ changes direction when $v(t) = 0$ and $a(t) \neq 0$. This happens @ $t = \frac{10}{3}$ and $t = 12$.

Speed:

Domain Interval	$v(t)$	$a(t)$	speed
$(0, \frac{10}{3})$	$-$	$+$	decreasing
$(\frac{10}{3}, \frac{23}{3})$	$+$	$+$	increasing
$(\frac{23}{3}, 12)$	$+$	$-$	decreasing
$(12, \infty)$	$-$	$-$	increasing

∴ The particle slows down when velocity & acceleration have opposite signs.

The particle speeds up when velocity & acceleration have the same signs.

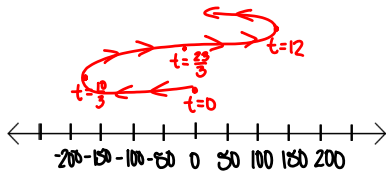
Direction:

Domain Interval	$v(t)$	direction
$(0, \frac{10}{3})$	$-$	left
$(\frac{10}{3}, 12)$	$+$	right
$(12, \infty)$	$-$	left

∴ The particle moves left when its velocity is negative and right when its velocity is positive.

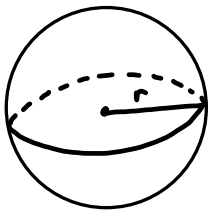
	0	$\frac{10}{3}$	$\frac{23}{3}$	12
$s(t)$	0	$\frac{4000}{27}$	$-\frac{529}{27}$	144
$v(t)$	-120	0	$\frac{169}{3}$	0
$a(t)$	46	26	0	-26

→ graph



Solve each related rate problem.

8) A spherical snowball is rolled in fresh snow, causing it to grow so that its radius increases at a rate of 3 in/sec. How fast is the volume of the snowball increasing when the radius is 7 in?



$\frac{dr}{dt} = 3 \text{ in/sec}$ $\frac{dV}{dt} = ?$

when $r = 7$ in

$V = \frac{4}{3}\pi r^3$

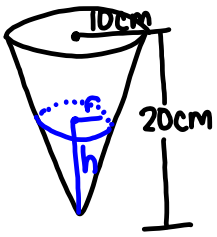
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(7)^2(3)$

$\frac{dV}{dt} = 4\pi(49)(3)$

$\frac{dV}{dt} = 588\pi \text{ in}^3/\text{sec}$

9) A conical paper cup is 20 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 9 cm?



$\frac{dh}{dt} = 2 \text{cm/sec}$

$\frac{dV}{dt} = ?$ when $h = 9$cm

$\frac{10}{20} = \frac{r}{h}$

$20r = 10h$

$r = \frac{1}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi(9)^2(2)$

$\frac{dV}{dt} = \frac{1}{2}\pi(81)$

$\frac{dV}{dt} = \frac{81\pi}{2} \text{ cm}^3/\text{sec}$