

Today's Plan:

Learning Target (standard): I will calculate the determinants of 2 x 2 and 3 x 3 matrices.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Solve using the elimination method.

$$\begin{array}{r}
 4(2x - y + 3z = 9) \\
 x + 4y + 4z = 5 \\
 -2(3x + 2y + 2z = 5)
 \end{array}
 \qquad
 \begin{array}{r}
 8x - 4y + 12z = 36 \\
 x + 4y + 4z = 5 \\
 -6x - 4y - 4z = -10
 \end{array}$$

$$\begin{array}{r}
 9x + 16z = 41 \\
 -5x = -5 \\
 \textcircled{x=1}
 \end{array}
 \qquad
 \begin{array}{r}
 9 + 16z = 41 \\
 16z = 32 \\
 \textcircled{z=2}
 \end{array}$$

$$3 + 2y + 4 = 5$$

$$2y + 7 = 5$$

$$2y = -2$$

$$\textcircled{y=-1}$$

independent
(1, -1, 2)

Solve using the elimination method.

$$\begin{array}{l} 2(3x - 2y + z = 2) \\ -1(2x + 3y + 2z = -6) \\ 2(3x - y + z = 0) \end{array} \quad \begin{array}{l} 6x - 4y + 2z = 4 \\ -2x - 3y - 2z = 6 \\ 6x - 2y + 2z = 0 \end{array}$$

$$\begin{array}{l} 4x - 7y = 10 \\ -1(4x - 5y = 6) \end{array} \quad \begin{array}{l} 4x - 7y = 10 \\ -4x + 5y = -6 \\ \hline -2y = 4 \end{array}$$

$$4x + 10 = 6$$

$$4x = -4$$

$$x = -1$$

$$y = -2$$

$$-3 + 2 + z = 0$$

$$-1 + z = 0$$

$$z = 1$$

independent
(-1, -2, 1)

Matrix: A rectangular array of numbers

$$\begin{bmatrix} 1 & -8 & 3 \\ 5 & 6 & 7 \\ 5 & 0 & -2 \\ 2 & & \end{bmatrix}$$

a_{ij} = element in row i ,
column j

$$a_{23} = 7$$

$$a_{31} = \frac{5}{2}$$

Element: Each number in the matrix

Rows: Horizontal numbers

Columns: Vertical numbers

Order of a Matrix: number of **rows** by the number of **columns**

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

rows \times columns

$$3 \times 3$$

$$23 \times 12$$

row 10, column 11

$$a_{10 \ 11}$$

Determinant of a 2 x 2 matrix:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

$$D =$$

$$\begin{vmatrix} 4 & -2 \\ 3 & 5 \end{vmatrix} = 4(5) - (-2)(3) \\ = 20 + 6$$

$$D = 26$$

Minor of a matrix:

- the determinant of a matrix after row i and column j have been removed

$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 4 & 1 \\ 2 & -5 & 0 \end{bmatrix}$$

 M_{ij}

$$M_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} = 0 - 6$$

$$M_{22} = -6$$

$$M_{13} = \begin{vmatrix} -3 & 4 \\ 2 & -5 \end{vmatrix} = 15 - 8$$

$$M_{13} = 7$$

Cofactor of a matrix:

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$\begin{bmatrix} 2 & 5 & -2 \\ -3 & -1 & 0 \\ -1 & 4 & -4 \end{bmatrix}$$

$$C_{12} = (-1)^{1+2} M_{12}$$

$$= -1 \cdot M_{12}$$

$$= -1 \begin{vmatrix} -3 & 0 \\ -1 & -4 \end{vmatrix}$$

$$= -1 (12 - 0)$$

$$C_{12} = -12$$

Determinant of a 3 x 3 matrix:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot C_{11} + a_{12} \cdot C_{12} + a_{13} \cdot C_{13}$$

Find the value of the determinant:

$$\begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 0 \\ 3 & -1 & 2 \end{vmatrix} = a_{11} \cdot C_{11} + a_{12} \cdot C_{12} + a_{13} \cdot C_{13}$$

$$= 2 \cdot (-1)^{1+1} M_{11} - 1 \cdot (-1)^{1+2} M_{12} - 3 \cdot (-1)^{1+3} M_{13}$$

$$= 2M_{11} + M_{12} - 3M_{13}$$

$$= 2 \begin{vmatrix} 2 & 0 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= 2(4-0) + (2-0) - 3(-1-6)$$

$$= 8+2+21$$

$$D = 31$$

Find the determinant of each matrix.

$$\begin{vmatrix} -2 & 3 & 1 \\ 4 & -2 & 0 \\ 1 & -2 & 3 \end{vmatrix} = a_{11} \cdot C_{11} + a_{12} \cdot C_{12} + a_{13} \cdot C_{13}$$

$$= -2(-1)^{1+1}M_{11} + 3(-1)^{1+2}M_{12} + 1(-1)^{1+3}M_{13}$$

$$= -2M_{11} - 3M_{12} + M_{13}$$

$$= -2 \begin{vmatrix} -2 & 0 \\ -2 & 3 \end{vmatrix} - 3 \begin{vmatrix} 4 & 0 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 4 & -2 \\ 1 & -2 \end{vmatrix}$$

$$= -2(-6 - 0) - 3(12 - 0) + (-8 + 2)$$

$$= 12 - 36 - 6$$

$$D = -30$$

Assignment:

Determinants Worksheet #1-8

* show ALL steps *