Today's Plan:

Learning Target (standard): I will find the average value of a function and describe its meaning.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Integration Practice (non-transcendental) #1-10

1) $\frac{5}{6}$

2) $-9\sqrt[3]{3}$

3) $\frac{6}{5}$

4) $-\frac{21}{200}$

5) $-\frac{1}{5}$

6) $-\frac{8}{3}$

7) $-\frac{3}{2\left(5x^5+1\right)^2}+C$

8) $\frac{5}{6}\left(5x^2-2\right)^6+C$

9) $\frac{9}{4}\sqrt[3]{\left(5x^2+3\right)^4}+C$

10) $2\sqrt{\left(4x^5-3\right)^3}+C$

Find the integral.

$$\int (3-x)^{10}\,dx$$

$u = 3 - x$

$du = -dx$

$-du = dx$

$\Rightarrow -\int u^{10}\,du$

$= -\frac{1}{11}u^{11} + C$

$\Rightarrow -\frac{1}{11}(3-x)^{11} + C$

Find the integral.

$$\int \sqrt{7x+9}\,dx$$

$u = 7x + 9$

$du = 7dx$

$\frac{1}{7}du = dx$

$\Rightarrow \frac{1}{7}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{7}\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C$

$= \frac{2}{21}u^{\frac{3}{2}} + C$

$\Rightarrow \frac{2}{21}\sqrt{(7x+9)^3} + C$

Find the integral.

$$\int \frac{x^3}{\left(1+x^4\right)^{\frac{1}{3}}}\,dx \Rightarrow \frac{1}{4}\int u^{-\frac{1}{3}}\,du$$

$u = 1 + x^4$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

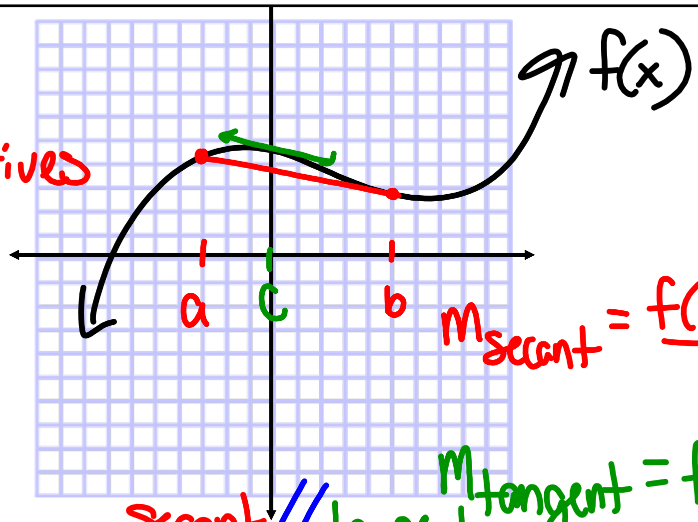
$$= \frac{1}{4}\left(\frac{3}{2}u^{\frac{2}{3}}\right) + C$$

$$= \frac{3}{8}u^{\frac{2}{3}} + C$$

$$\Rightarrow \frac{3}{8}\left(1+x^4\right)^{\frac{2}{3}} + C$$

MVT

Derivatives



$$m_{secant} = \frac{f(b)-f(c)}{b-a}$$

$$m_{tangent} = f'(c)$$

secant // tangent

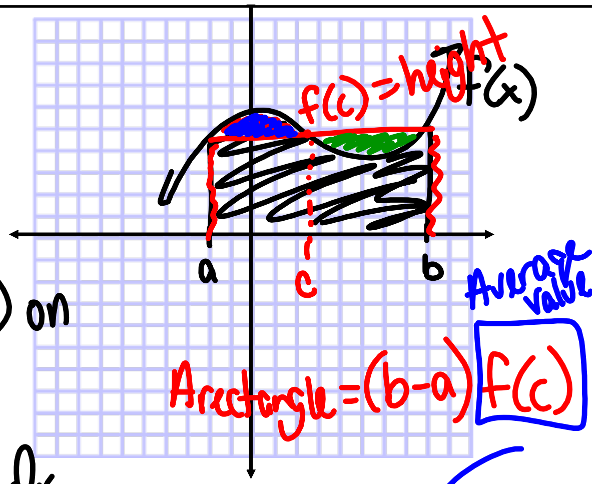
$$\frac{f(b)-f(c)}{b-a} = f'(c)$$

$$f(b)-f(a) = (b-a)f'(c)$$

MVT Integrals

Area under f(x) on [a,b]

$$A=\int_a^b f(x)\,dx$$



?

$$\int_a^b f(x)\,dx=(b-a)f(c)$$

$$\frac{1}{b-a}\int_a^b f(x)\,dx=f(c) \leftarrow \text{height}$$

"average value"

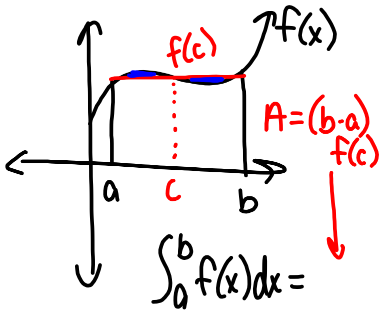
Find the average value of the function on the given interval. Write the meaning of the answer.

$f(x)=x^2;[2,4]$ $\quad f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$

$$f(c)=\frac{1}{4-2}\int_2^4 x^2\,dx$$
$$=\frac{1}{2}\left(\frac{1}{3}x^3\right)\Big|_2^4$$
$$=\frac{1}{6}\left(4^3-2^3\right)$$
$$=\frac{1}{6}(64-8)$$

$$f(c)=\frac{28}{3}$$

∴ The area under f(x) = x^2 on [2,4] will be equal to the area of the rectangle with length (4-2) and height f(c) = $^{28}/_3$. So, the area will be $^{56}/_3$ u^2.



Find the average value of the function on the given interval.
Explain the meaning of the average value.

$$f(x) = 2x^3;[1,3]$$

$$f(c) = \frac{1}{3-1}\int_1^3 2x^3\,dx$$

$$= \frac{1}{2}\left(\frac{1}{2}x^4\right)\Big|_1^3$$

$$= \frac{1}{4}x^4\Big|_1^3$$

$$= \frac{1}{4}(3^4 - 1^4)$$

$$= \frac{1}{4}(81-1)$$

$$= \frac{1}{4}(80)$$

$$f(c) = 20$$

$\therefore$ The area under $f(x) = 2x^3$ on [1,3] will be equal to the area of the rectangle with length (3-1) and height f(c) = 20. So, the area will be 40 u^2.

Evaluate the integral and then find the derivative.

$$\int_1^x t^2\,dt = \frac{1}{3}t^3\Big|_1^x$$

$$= \frac{1}{3}x^3 - \frac{1}{3}$$

$$\frac{d}{dx}\int_1^x t^2\,dt = \frac{d}{dx}\left(\frac{1}{3}x^3 - \frac{1}{3}\right)$$

$$= x^2$$

Evaluate the integral and then find the derivative.

$$\int_2^x \left(1-t^3\right)dt = \left(t - \frac{1}{4}t^4\right)\Big|_2^x$$

$$= \left(x - \frac{1}{4}x^4\right) - \left(2 - \frac{1}{4}(2)^4\right)$$

$(2-4)$

$$= x - \frac{1}{4}x^4 + 2$$

$$\frac{d}{dx}\int_2^x \left(1-t^3\right)dt = \frac{d}{dx}\left(x - \frac{1}{4}x^4 + 2\right)$$

$$= 1 - x^3$$

Evaluate the integral and then find the derivative.

$$\int_2^{x^2} t^2\,dt = \frac{1}{3}x^3 \Big|_2^{x^2}$$

$$= \frac{1}{3}\left(x^2\right)^3 - \frac{1}{3}(2)^3$$

$$= \frac{1}{3}x^6 - \frac{8}{3}$$

$$\frac{d}{dx}\int_2^{x^2} t^2\,dt = \frac{d}{dx}\left(\frac{1}{3}x^6 - \frac{8}{3}\right)$$

$$= 2x^5$$

t^2

$\left(x^2\right)^2 \cdot 2x$

$x^4 \cdot 2x$

$2x^5$

Fundamental Theorem of Calculus:

- First Fundamental Theorem

$$\int_a^b f(x)dx = F(b) - F(a)$$

- Second Fundamental Theorem

$$\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))\bullet g'(x)$$

Find the derivative.

$$\frac{d}{dx}\int_1^x \cos t\,dt = \cos x \cdot 1$$

$$= \cos x$$

Find the derivative.

$$\frac{d}{dx}\int_0^{3x^4}\left(t+4t^2\right)dt = \left(3x^4+4(3x^4)^2\right)\cdot 12x^3$$

$$= \left(3x^4+36x^8\right)\cdot 12x^3$$

$$= 12x^3\left(3x^4+36x^8\right)$$

$$= 36x^7\left(1+12x^4\right)$$

Assignment:

For 1 - 2, find the average value of the function. Explain the meaning of the result.

1) $f(x)=\sqrt{x};[0,16]$

2) $f(x)=\sqrt{1-x};[-1,1]$

For 3 - 6, find the derivative.

3) $\frac{d}{dx}\int_1^x \sin^2 t\,dt$

4) $\frac{d}{dx}\int_5^{3x}\left(t^2-t\right)dt$

5) $\frac{d}{dx}\int_0^{x^2}|t|\,dt$

6) $\frac{d}{dx}\int_1^x -2\cos t\,dt$

7) Estimate the area under $f(x) = 3x^3 + x$ on [1,3] using:

a) 4 inscribed rectangles
b) 4 circumscribed rectangles
c) 4 trapezoids

Answers to Assignment:

1) $f(c) = \frac{8}{3}$

2) $f(c) = \frac{2\sqrt{2}}{3}$

3) $\sin^2 x$

4) $27x^2 - 9x$

5) $2x^3$

6) $-2\cos x$

7a) $\frac{91}{2}u^2$

b) $\frac{171}{2}u^2$

c) $\frac{131}{2}u^2$

Assignment:

Integration Practice

#1-14