Today's Plan:

Learning Target (standard): I will solve real-world rates of change application problems. I will describe the motion of a particle.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make neccessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

The volume of a spherical balloon after t hours is given by $v(t) = \frac{4}{3}\pi\,(4t - 1)^3$ cm^3. Find the rate of change of each of the following at 4 hours:

1. volume
2. radius
3. surface area

① $V'(t) = 4\pi(4t-1)^2(4)$

$V'(4) = 4\pi(15)^2(4)$

$V'(4) = 3600\pi$ cm^3/hr

② $r(t) = 4t-1$

$r'(t) = 4$

$r'(4) = 4$ cm/hr

③ $SA(t) = 4\pi(4t-1)^2$

$SA'(t) = 8\pi(4t-1)(4)$

$SA'(t) = 32\pi(4t-1)$

$SA'(4) = 32\pi(15)$

$SA'(4) = 480\pi$ cm^2/hr

If the position of a particle is given by $s(t) = 2t^3 - 21t^2 + 60t + 3$, when $t > 0$ seconds, find the particle's velocity and acceleration. Find the interval of time during which the particle is speeding up. Find the distance the particle travels between 2 and 5 seconds.

① $v(t) = s'(t)$

$v(t) = 6t^2 - 42t + 60$

$0 = 6(t^2 - 7t + 10)$

$0 = 6(t-5)(t-2)$

$t = 2, 5$ seconds

$a(t) = v'(t) = s''(t)$

$a(t) = 12t - 42$

$0 = 6(2t - 7)$

$t = \frac{7}{2}$ seconds

∴ The particle changes direction when the velocity is 0 and acceleration is not 0. The particle will change direction @ time 2 and 5 seconds.

②

Domain Interval	v(t)	a(t)	Speed
$(0,2)$	+	−	decreasing
$(2,\frac{7}{2})$	−	−	increasing
$(\frac{7}{2},5)$	−	+	decreasing
$(5,\infty)$	+	+	increasing

∴ The particle will speed up when the velocity & acceleration have the same sign. This happens when $2 < t < \frac{7}{2}$ seconds and $t > 5$ seconds.

③ no change of direction between 2 & 5 seconds

$d = |s(5) - s(2)|$

$d = |28 - 55|$

$d = 27$ units

How far does a particle travel between the eighth and tenth seconds if its position function is $s(t) = t^2 - 6t$? Between the second and fourth seconds?

$v(t) = 2t - 6$ $\quad a(t) = 2$

$0 = 2(t-3)$

$t = 3$

∴ The particle changes direction when velocity is 0 and acceleration is not 0. This happens @ $t = 3$ seconds.

① distance between 8 & 10 seconds – no direction change

$d = |s(10) - s(8)|$

$= |40 - 16|$

$d = 24$ units

② distance between 2 & 4 seconds – change of direction @ $t = 3$ seconds

$d = |s(3) - s(2)| + |s(4) - s(3)|$

$= |-9 + 8| + |-8 + 9|$

$= 1 + 1$

$d = 2$ units

The position function of a point on a coordinate line is given by $s(t) = 3t^2 - 12t + 1$, where t is measured in seconds and $s(t)$ in centimeters. Describe the motion of the particle during the interval [0,5].

① $v(t) = 6t - 12$ $a(t) = 6$

$0 = 6(t-2)$

$t = 2$ sec

∴ The particle changes direction when velocity is 0 and acceleration is not 0. This happens @ $t = 2$ seconds.

②

Domain Interval	$v(t)$	$a(t)$	Speed
$(0,2)$	$-$	$+$	decreasing
$(2,5)$	$+$	$+$	increasing

∴ The particle is slowing down when the velocity & acceleration have opposite signs.

This happens when $0 < t < 2$ seconds. The particle speeds up when the velocity and acceleration have the same signs. This happens when $2 < t \leq 5$ seconds.

③

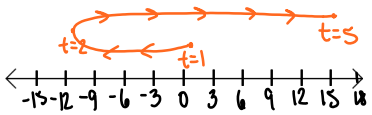
Domain Interval	6	$t-2$	$v(t)$	Direction
$(0,2)$	$+$	$-$	$-$	left
$(2,5)$	$+$	$+$	$+$	right

∴ The particle travels to the left when its velocity is negative. This happens when $0 < t < 2$ seconds. The particle travels to the right when its velocity is positive. This happens when $2 < t \leq 5$ seconds.

④

	0	2	5
$s(t)$	1	-11	16
$v(t)$	-12	0	18
$a(t)$	6	6	6

⑤



Assignment:

Rate of Change Worksheet #1-4

Please watch video and copy notes on related rates

Systematic Procedure for Solving Related Rate Problems

1) Let t denote the elapsed time. Draw a diagram that is valid for all $t > 0$. Label those quantities whose values do not change as t increases with their given constant values.

2) State what is given about the variables and what information is wanted about them. This information will be in the form of derivatives with respect to t.

3) Write an equation relating variables that is valid at all times $t > 0$, not just at some particular instant.

4) Differentiate the equation found in the previous step implicitly with respect to time. The resulting equation, containing derivatives with respect to t, is true for all $t > 0$.

Systematic Procedure for Solving Related Rate Problems

5) Substitute in the equation found in step 4 all data that are valid *at the particular instant* for which the answer to the problem is required.

6) Solve for the desired derivative.