A 15 foot ladder is resting against a wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of $\frac{1}{4}$ ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?

\[
\begin{align*}
X^2 + y^2 &= 225 \\
49 + y^2 &= 225 \\
y^2 &= 176 \\
y &= \sqrt{176} = \sqrt{14.44} = 4\sqrt{11} \\
\end{align*}
\]

\[
\frac{dx}{dt} = -\frac{1}{4} \text{ ft/sec}
\]

\[
\begin{align*}
x^2 + y^2 &= 15^2 \\
2x\frac{dx}{dt} + 2y\frac{dy}{dt} &= 0 \\
2(7)(-\frac{1}{4}) + 2(\sqrt{176})\frac{dy}{dt} &= 0 \\
-\frac{7}{2} + 2\sqrt{176}\frac{dy}{dt} &= 0 \\
2\sqrt{176}\frac{dy}{dt} &= \frac{7}{2} \\
\frac{dy}{dt} &= \frac{7}{2\sqrt{176}} = \frac{7}{16\sqrt{11}} \text{ ft/sec}
\end{align*}
\]
Air is being pumped into a spherical balloon at a rate of $5 \text{ cm}^3/\text{min}$. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm.

$$\frac{dV}{dt} = 5 \text{ cm}^3/\text{min}$$

$$\frac{dr}{dt} = ?$$

When $d = 20 \text{ cm} \Rightarrow r = 10 \text{ cm}$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$5 = 4\pi (10)^2 \frac{dr}{dt}$$

$$5 = 400\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{80\pi} \text{ cm/min}$$
Two people are at an elevator. At the same time, one person starts to walk away from the elevator at a rate of 2 ft/sec and the other person starts going up in the elevator at a rate of 7 ft/sec. What rate is the distance between the two people changing 15 seconds later?

\[ x = 2(15) = 30 \text{ ft} \]
\[ y = 7(15) = 105 \text{ ft} \]

\[ x^2 + y^2 = z^2 \]
\[ 30^2 + 105^2 = z^2 \]
\[ 900 + 11025 = z^2 \]
\[ z^2 = 11925 \]
\[ z = \sqrt{11925}, -\sqrt{11925} \]
\[ z = 15\sqrt{53} \]

\[ x = 30 \text{ ft} \]
\[ y = 105 \text{ ft} \]
\[ z = 15\sqrt{53} \text{ ft} \]

\[ \frac{dx}{dt} = 2 \text{ ft/sec} \]
\[ \frac{dy}{dt} = 7 \text{ ft/sec} \]

\[ x^2 + y^2 = z^2 \]
\[ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt} \]
\[ 2(30)(2) + 2(105)(7) = 2(15\sqrt{53}) \frac{dz}{dt} \]
\[ 60 + 105 \cdot 7 = 2(15\sqrt{53}) \frac{dz}{dt} \]
\[ 795 = 30\sqrt{53} \frac{dz}{dt} \]
\[ \frac{795}{30\sqrt{53}} = \frac{dz}{dt} \]
\[ \frac{53}{153} = \frac{dz}{dt} \]

\[ \frac{dz}{dt} = \sqrt{53} \text{ ft/sec} \]
A thin sheet of ice is in the form of a circle. If the ice is melting in such a way that the area of the sheet is decreasing at a rate of 0.5 m$^2$/sec, at what rate is the radius decreasing when the area of the sheet is 12 m$^2$?

\[
\frac{dA}{dt} = -\frac{1}{2} \text{ m}^2/\text{sec}
\]

\[
\frac{dr}{dt} = ? \quad \text{when } A = 12 \text{ m}^2
\]

\[
A = \pi r^2
\]

\[
12 = \pi r^2
\]

\[
\frac{12}{\pi} = r^2
\]

\[
r = \frac{2\sqrt{3}}{\pi}
\]

\[
\frac{dA}{dt} = 2\pi r \frac{dr}{dt}
\]

\[
-\frac{1}{2} = 2\pi \left( \frac{2\sqrt{3}}{\pi} \right) \frac{dr}{dt}
\]

\[
-\frac{1}{2} = 4\sqrt{3} \frac{dr}{dt}
\]

\[
-\frac{1}{8\sqrt{3}} = \frac{dr}{dt}
\]

\[
\frac{dr}{dt} = -\frac{\sqrt{3}\pi}{24\pi} \text{ m/sec}
\]
A tank of water in the shape of a cone is being filled up with water at a rate of 12 m³/sec. The base radius of the tank is 26 meters and the height of the tank is 8 meters. At what rate is the depth of the water changing when the radius of the top of the water is 10 meters?