Today's Plan:

Learning Target (standard): I will find the area under a curve using Riemann sums and define the area as a definite integral.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

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QUIZ Monday!

1) 30

2) $9\sqrt{2}$

3) -12

4) -7

5) 2

6) 0

7) 78

8) 42

9) $-\frac{291}{2}$

10) 291

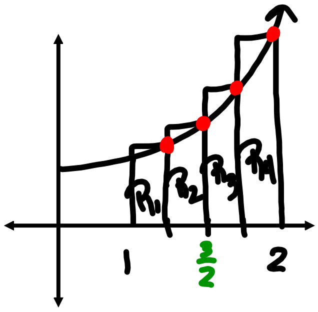
11) $-\frac{14\sqrt{5}}{3}$

12) 57

13) $\frac{215}{6}$

14) $\frac{41}{6}$

Estimate the area under $y = x^3 + 1$ on the interval [1,2] using 4 circumscribed rectangles.



$\Delta x = \frac{2-1}{4} = \frac{1}{4}$

$A_{R_1} = \frac{1}{4}\left(f\left(\frac{5}{4}\right)\right)$

$= \frac{1}{4}\left(\frac{125}{64} + 1\right)$

$= \frac{1}{4}\left(\frac{125+64}{64}\right)$

$A_{R_1} = \frac{189}{256}\, u^2$

$A_{R_2} = \frac{1}{4}\left(f\left(\frac{3}{2}\right)\right)$

$= \frac{1}{4}\left(\frac{27}{8} + \frac{8}{8}\right)$

$A_{R_2} = \frac{35}{32}\, u^2$

$A_{R_3} = \frac{1}{4}\left(f\left(\frac{7}{4}\right)\right)$

$= \frac{1}{4}\left(\frac{343}{64} + \frac{64}{64}\right)$

$A_{R_3} = \frac{407}{256}\, u^2$

$A_{R_4} = \frac{1}{4}(f(2))$

$= \frac{1}{4}(9)$

$A_{R_4} = \frac{9}{4}\, u^2$

$A \approx \frac{189}{256} + \frac{35}{32} + \frac{407}{256} + \frac{9}{4}$

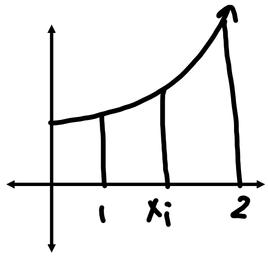
$\approx \frac{596}{256} + \frac{107}{32}$

$\approx \frac{596+856}{256}$

$\approx \frac{1452}{256}$

$A \approx \frac{363}{64}\, u^2$

Find the area under $y = x^3 + 1$ on the interval [1,2].



$\Delta x = \frac{2-1}{n} = \frac{1}{n}$

$x_0 = 1$

$x_1 = 1 + \Delta x = 1 + \frac{1}{n}$

$x_2 = 1 + 2\Delta x = 1 + \frac{2}{n}$

$\vdots$

$x_i = 1 + i\Delta x = 1 + \frac{i}{n}$

$\vdots$

$x_n = 1 + n\Delta x = 1 + n\left(\frac{1}{n}\right) = 1 + 1 = 2$ ✓

$$A_{R_i} = \Delta x \cdot f(x_i)$$
$$= \frac{1}{n}\left[\left(1+\frac{i}{n}\right)^3 + 1\right]$$
$$= \frac{1}{n}\left[1 + \frac{3i}{n} + \frac{3i^2}{n^2} + \frac{i^3}{n^3} + 1\right]$$
$$= \frac{1}{n}\left[2 + \frac{3i}{n} + \frac{3i^2}{n^2} + \frac{i^3}{n^3}\right]$$
$$A_{R_i} = \frac{2}{n} + \frac{3i}{n^2} + \frac{3i^2}{n^3} + \frac{i^3}{n^4}$$

$$A = \sum_{i=1}^{n} A_{R_i} = \sum_{i=1}^{n}\left(\frac{2}{n} + \frac{3i}{n^2} + \frac{3i^2}{n^3} + \frac{i^3}{n^4}\right)$$
$$= \sum_{i=1}^{n}\frac{2}{n} + \sum_{i=1}^{n}\frac{3i}{n^2} + \sum_{i=1}^{n}\frac{3i^2}{n^3} + \sum_{i=1}^{n}\frac{i^3}{n^4}$$
$$= 2 + \frac{3}{n^2}\sum_{i=1}^{n} i + \frac{3}{n^3}\sum_{i=1}^{n} i^2 + \frac{1}{n^4}\sum_{i=1}^{n} i^3$$

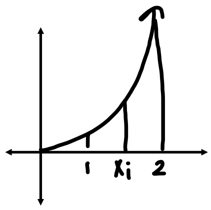
$\cancel{n^2}(n^2+2n+1)$

$$= 2 + \frac{3}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{2}\right) + \frac{\cancel{3}^{\,1}}{n^{\cancel{3}\,2}}\left(\frac{\cancel{n}^{\,1}(n+1)(2n+1)}{\cancel{6}\,2}\right) + \frac{1}{n^{\cancel{4}\,2}}\left[\frac{n(n+1)}{2}\right]^2$$
$$= 2 + \frac{3}{2n}(n+1) + \frac{1}{2n^2}(2n^2+3n+1) + \frac{1}{4n^2}(n^2+2n+1)$$
$$= 2 + \frac{3}{2} + \frac{3}{2n} + 1 + \frac{3}{2n} + \frac{1}{2n^2} + \frac{1}{4} + \frac{1}{2n} + \frac{1}{4n^2}$$
$$= \frac{19}{4} + \frac{7}{2n} + \frac{1}{2n^2} + \frac{1}{4n^2}$$

$$A = \lim_{n\to\infty}\left(\frac{19}{4} + \frac{7}{2n} + \frac{1}{2n^2} + \frac{1}{4n^2}\right)$$
$$= \frac{19}{4} + 0 + 0 + 0$$
$$A = \frac{19}{4}\,u^2$$

Riemann Sums Review **March 1, 2024**

Find the area under $f(x) = x^3$ on the interval [1,2]. Write as a definite integral. Use this to evaluate $\int_1^2 (x^3+1)\,dx$



$\Delta x = \frac{2-1}{n} = \frac{1}{n}$

$x_0 = 1$

$x_1 = 1 + \Delta x = 1 + \frac{1}{n}$

$x_2 = 1 + 2\Delta x = 1 + \frac{2}{n}$

$\vdots$

$x_i = 1 + i\Delta x = 1 + \frac{i}{n}$

$\vdots$

$x_n = 1 + n\Delta x = 1 + n\left(\frac{1}{n}\right) = 1 + 1 = 2 \checkmark$

$A_{R_i} = \Delta x \cdot f(x_i)$

$= \frac{1}{n}\left[\left(1+\frac{i}{n}\right)^3\right]$

$= \frac{1}{n}\left[1 + \frac{3i}{n} + \frac{3i^2}{n^2} + \frac{i^3}{n^3}\right]$

$A_{R_i} = \frac{1}{n} + \frac{3i}{n^2} + \frac{3i^2}{n^3} + \frac{i^3}{n^4}$

$$A = \sum_{i=1}^{n} A_{R_i} = \sum_{i=1}^{n}\left(\frac{1}{n} + \frac{3i}{n^2} + \frac{3i^2}{n^3} + \frac{i^3}{n^4}\right)$$

$$= \sum_{i=1}^{n}\frac{1}{n} + \sum_{i=1}^{n}\frac{3i}{n^2} + \sum_{i=1}^{n}\frac{3i^2}{n^3} + \sum_{i=1}^{n}\frac{i^3}{n^4}$$

$$= 1 + \frac{3}{n^2}\sum_{i=1}^{n} i + \frac{3}{n^3}\sum_{i=1}^{n} i^2 + \frac{1}{n^4}\sum_{i=1}^{n} i^3$$

$\cancel{n^2}(n^2+2n+1)$

$$= 1 + \frac{3}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{2}\right) + \frac{\cancel{3}1}{n^{\cancel{3}}}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}2}\right) + \frac{1}{n^{\cancel{4}2}}\left[\frac{n(n+1)}{2}\right]^2 \; 4$$

$$= 1 + \frac{3}{2n}(n+1) + \frac{1}{2n^2}(2n^2+3n+1) + \frac{1}{4n^2}(n^2+2n+1)$$

$$= 1 + \frac{3}{2} + \frac{3}{2n} + 1 + \frac{3}{2n} + \frac{1}{2n^2} + \frac{1}{4} + \frac{1}{2n} + \frac{1}{4n^2}$$

$$= \frac{15}{4} + \frac{7}{2n} + \frac{1}{2n^2} + \frac{1}{4n^2}$$

$$A = \lim_{n\to\infty}\left(\frac{15}{4} + \frac{7}{2n} + \frac{1}{2n^2} + \frac{1}{4n^2}\right)$$

$$= \frac{15}{4} + 0 + 0 + 0$$

$$A = \frac{15}{4}u^2 \rightarrow \int_1^2 x^3\,dx$$

$$\int_1^2 (x^3+1)\,dx$$

$$= \int_1^2 x^3\,dx + \int_1^2 1\,dx$$

$$= \frac{15}{4} + 1(2-1)$$

$$= \frac{15}{4} + 1$$

$$= \frac{19}{4}$$

$$\boxed{\int_0^2 x^3 dx = 4}$$

$$\boxed{\int_0^2 x dx} = 2$$

$$\int_0^2 \left(-2x^3 + 4x - 3\right) dx =$$

$$= \int_0^2 -2x^3 dx + \int_0^2 4x dx - \int_0^2 3dx$$

$$= -2\boxed{\int_0^2 x^3 dx} + 4\boxed{\int_0^2 x dx} - \int_0^2 3dx$$

$$= -2(4) + 4(2) - 3(2-0)$$

$$= -8 + 8 - 6$$

$$= -6$$

$$\boxed{\int_0^2 x^3\,dx = 4}$$

$$\boxed{\int_0^2 x\,dx = 2}$$

$$\int_0^2 \left(-8x^3 + 6x - 5\right)dx =$$

$$= \int_0^2 -8x^3\,dx + \int_0^2 6x\,dx - \int_0^2 5\,dx$$

$$= -8\boxed{\int_0^2 x^3\,dx} + 6\boxed{\int_0^2 x\,dx} - \int_0^2 5\,dx$$

$$= -8(4) + 6(2) - 5(2-0)$$

$$= -32 + 12 - 10$$

$$= -30$$

Review: *QUIZ Monday!*

1) Find the area under $y = x^2 + 3$ on the interval [1,3]. $A = \frac{44}{3}u^2$

2) Find the area under $y = 2x^3$ on the interval [0,4]. $A = 128u^2$

3) Find the area under $f(x) = 4x^3 + 3x^2$ on the interval $[{}^{-3}/_{4},0]$.

$A = \frac{27}{256}u^2$

4) Find the area under $f(x) = x^2 + 3x$ on the interval [1,4].

$A = \frac{87}{2}u^2$

5) Find the area under $y = 4x^3 + 3x^2 + 2x + 1$ on the interval [2,5]. $A = 750u^2$

6) Estimate the area under $y = 4 - x^2$ on the interval [-1,1] using:

a. 4 inscribed rectangles $A = \frac{27}{4}u^2$
b. 4 circumscribed rectangles $A = \frac{31}{4}u^2$
c. 4 trapezoids $A = \frac{29}{4}u^2$

7) Find the area under y = 8 from x = -3 to x = 4 using definite integrals. $A = 56u^2$