Today's Plan:

Learning Target (standard): I will find the volume of a solid of revolution.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

p.274 #1-8

1) $V = \frac{2\pi}{3}u^3$

2) $V = 8\pi u^3$

3) $v = 2\pi u^3$

4) $V = \frac{2\pi}{3}u^3$

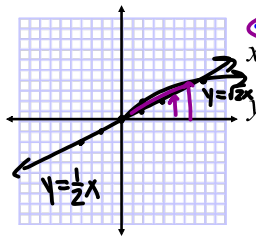
5) $V = \frac{512\pi}{15}u^3$

6) $V = \frac{128\pi}{7}u^3$

7) $V = \frac{64\pi}{15}u^3$

8) $V = \frac{\pi}{24}u^3$

Find the volume of the solid:



about x-axis

$x-2y=0$

$y^2-2x=0$

$-2y=-x$

$y=\frac{1}{2}x$

$y^2=2x$

$y=\sqrt{2x}$

x	y
0	0
2	2
8	4

$(\frac{1}{2}x)^2=(\sqrt{2x})^2$

$\frac{1}{4}x^2=2x$

$\frac{1}{4}x^2-2x=0$

$\frac{1}{4}x(x-8)=0$

$x=0,8$

$V_{outside}=\pi(\sqrt{2x})^2\,dx$

$V_{inside}=\pi(\frac{1}{2}x)^2\,dx$

$V=\pi\int_0^8\left[(\sqrt{2x})^2-(\frac{1}{2}x)^2\right]dx$

$=\pi\int_0^8(2x-\frac{1}{4}x^2)\,dx$

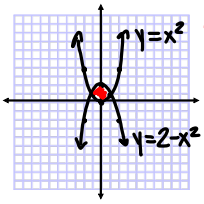
$=\pi(x^2-\frac{1}{12}x^3)\Big|_0^8$

$=\pi\left[(64-\frac{128}{3})-0\right]$

$=\pi(\frac{192-128}{3})$

$V=\frac{64\pi}{3}u^3$

Find the volume of the solid:



about x-axis

$y=x^2$

$y=2-x^2$

$x^2=2-x^2$

$2x^2-2=0$

$2(x^2-1)=0$

$2(x+1)(x-1)=0$

$x=-1,1$

$V_{outside}=\pi(2-x^2)^2\,dx$

$V_{inside}=\pi(x^2)^2\,dx$

$V=\pi\int_{-1}^{1}\left[(2-x^2)^2-(x^2)^2\right]dx$

$=\pi\int_{-1}^{1}(4-4x^2+x^4-x^4)\,dx$

$=\pi\int_{-1}^{1}(4-4x^2)\,dx$

$=\pi(4x-\frac{4}{3}x^3)\Big|_{-1}^{1}$

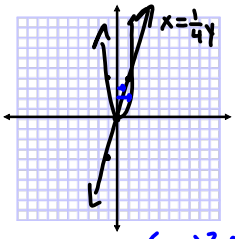
$=\pi\left[(4-\frac{4}{3})-(-4+\frac{4}{3})\right]$

$=\pi(8-\frac{8}{3})$

$=\pi(\frac{24-8}{3})$

$V=\frac{16\pi}{3}u^3$

Find the volume of the solid:



about y-axis

$y = 4x$ $x=\frac{1}{4}y$

$y = 4x^2$

$x^2=\frac{1}{4}y$

$x=\frac{1}{2}\sqrt{y}$

$(\frac{1}{4}y)=(\frac{1}{2}\sqrt{y})^2$

$\frac{1}{16}y^2=\frac{1}{4}y$

$\frac{1}{16}y^2-\frac{1}{4}y=0$

$\frac{1}{16}y(y-4)=0$

$y=0,4$

$V_{outside}=\pi(\frac{1}{2}\sqrt{y})^2\,dy$

$V_{inside}=\pi(\frac{1}{4}y)^2\,dy$

$$V=\pi\int_0^4\left[(\tfrac{1}{2}\sqrt{y})^2-(\tfrac{1}{4}y)^2\right]dy$$

$$=\pi\int_0^4\left(\tfrac{1}{4}y-\tfrac{1}{16}y^2\right)dy$$

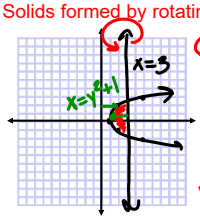
$$=\pi\left(\tfrac{1}{8}y^2-\tfrac{1}{48}y^3\right)\Big|_0^4$$

$$=\pi\left(\tfrac{1}{8}(16)-\tfrac{1}{48}(64)\right)$$

$$=\pi\left(2-\tfrac{4}{3}\right)$$

$$V=\frac{2\pi}{3}u^3$$

Solids formed by rotating about an alternate axis



about x = 3

$x = y^2+1$

$x = 3$

x	y
3	-2
2	-1
1	0
2	1
3	2

about x = -3

*parallel to y-axis → dy

$y^2+1=3$

$y^2=2$

$y=\sqrt{2},-\sqrt{2}$

$V=\pi r^2h$

$V=\pi(3-(y^2+1))^2\,dy$

$$V=\pi\int_{-\sqrt{2}}^{\sqrt{2}}(3-y^2-1)^2\,dy$$

$$=\pi\int_{-\sqrt{2}}^{\sqrt{2}}(2-y^2)^2\,dy$$

$$=\pi\int_{-\sqrt{2}}^{\sqrt{2}}(4-4y^2+y^4)\,dy$$

$$=\pi\left(4y-\tfrac{4}{3}y^3+\tfrac{1}{5}y^5\right)\Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$=\pi\left[\left(4\sqrt{2}-\tfrac{8\sqrt{2}}{3}+\tfrac{4\sqrt{2}}{5}\right)-\left(-4\sqrt{2}+\tfrac{8\sqrt{2}}{3}-\tfrac{4\sqrt{2}}{5}\right)\right]$$

$$=\pi\left(8\sqrt{2}-\tfrac{16\sqrt{2}}{3}+\tfrac{8\sqrt{2}}{5}\right)$$

$$=\pi\left(\tfrac{120\sqrt{2}-80\sqrt{2}+24\sqrt{2}}{15}\right)$$

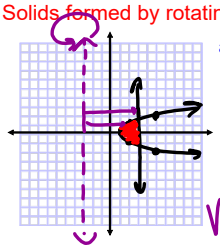
$$V=\frac{64\sqrt{2}\pi}{15}u^3$$

Solids formed by rotating about an alternate axis

about x = 3 about x = -3

$x = y^2 + 1$

$x = 3$



$V_{outside} = \pi(6)^2\,dy$

$V_{inside} = \pi(3+(y^2+1))^2\,dy$

$V = \pi\int_{-\sqrt{2}}^{\sqrt{2}} \left[6^2 - (3+y^2+1)^2\right] dy$

$= \pi\int_{-\sqrt{2}}^{\sqrt{2}} \left(36 - (4+y^2)^2\right) dy$

$= \pi\int_{-\sqrt{2}}^{\sqrt{2}} \left(36 - (16+8y^2+y^4)\right) dy$

$= \pi\int_{-\sqrt{2}}^{\sqrt{2}} (20 - 8y^2 - y^4)\,dy$

$= \pi\left(20y - \frac{8}{3}y^3 - \frac{1}{5}y^5\right)\Big|_{-\sqrt{2}}^{\sqrt{2}}$

$= \pi\left[\left(20\sqrt{2} - \frac{16\sqrt{2}}{3} - \frac{4\sqrt{2}}{5}\right) - \left(-20\sqrt{2} + \frac{16\sqrt{2}}{3} + \frac{4\sqrt{2}}{5}\right)\right]$

$= \pi\left(40\sqrt{2} - \frac{32\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}\right)$

$= \pi\left(\frac{600\sqrt{2} - 160\sqrt{2} - 24\sqrt{2}}{15}\right)$

$V = \frac{416\sqrt{2}\pi}{15}\ u^3$

Assignment:

p.274 #13-14

p.274 #13-14

$13a) V = \frac{512\pi}{15} u^3$

$b) V = \frac{832\pi}{15} u^3$

$c) V = \frac{128\pi}{3} u^3$

$14a) V = \frac{256\pi}{15} u^3$

$b) V = \frac{192\pi}{5} u^3$

$c) V = \frac{40\pi}{3} u^3$