

## Today's Plan:

**Learning Target (standard):** I will find determinants of matrices.

**Students will:** Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work and complete practice problems on functions.

**Teacher will:** Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of review concepts and assign students assessment problems over functions.

**Assessment:** Board work, homework check and homework assignment

**Differentiation:** Students will work at the board, go over and correct homework at their seats, actively engage in review problems, practice review concepts with the aid of other students and the teacher and complete homework assignment.

Solve each system using the elimination method:

$$\begin{array}{l}
 10y - 16 = 12x \\
 -8x = -4y
 \end{array}
 \quad
 \begin{array}{l}
 2(12x - 10y = -16) \\
 3(-8x + 4y = 0)
 \end{array}
 \quad
 \begin{array}{r}
 24x - 20y = -32 \\
 -24x + 12y = 0 \\
 \hline
 -8y = -32 \\
 y = 4
 \end{array}$$

independent  
 $(2, 4)$

$$\begin{array}{l}
 -8x = -4(4) \\
 -8x = -16 \\
 x = 2
 \end{array}$$

Solve each system using the elimination method:

$$\begin{array}{rcl} (5a - 4b - 6c = -1) \cdot 3 & 15a - 12b - 18c = -3 & \\ (-4a + 6b - 5c = 26) \cdot 2 & -8a + 12b - 10c = 52 & > + \\ (-a + b + 5c = -7) \cdot 12 & 12a - 12b - 60c = 84 & > + \end{array}$$

$$\begin{array}{rcl} 4(7a - 28c = 49) & 28a - 112c = 196 & \\ -7(4a - 70c = 136) & -28a + 490c = -952 & \\ \hline & 378c = -756 & \end{array}$$

$$\begin{array}{rcl} \text{independent} & 7a - 28(-2) = 49 & c = -2 \\ (-1, 2, -2) & 7a + 56 = 49 & \\ & 7a = -7 & \\ & a = -1 & \end{array}$$

$$\begin{array}{l} -(-1) + b + 5(-2) = -7 \\ 1 + b - 10 = -7 \\ b - 9 = -7 \\ b = 2 \end{array}$$

Matrix: A rectangular array of numbers

$$\begin{bmatrix} 1 & -8 & 3 \\ 5 & 6 & 7 \\ 5 & 0 & -2 \\ 2 & & \end{bmatrix}$$

← rows

← elements

↑ columns

Element: Each **number** in the matrix

Rows: Horizontal numbers

Columns: Vertical numbers

Order of a Matrix: number of **rows** by the number of **columns**

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

3 rows x 3 columns

$a_{ij}$   
 ↗ row #      ↖ column #

$a_{12}$  = "element in row 1, column 2"

**Determinant** of a 2 x 2 matrix:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

$$\begin{vmatrix} 4 & -2 \\ 3 & 5 \end{vmatrix} = 4(5) - (-2)(3)$$

$$= 20 + 6$$

$$D = 26$$

## Minor of a matrix: $M_{ij}$

- the determinant of a matrix after row  $i$  and column  $j$  have been removed

$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & 4 & 1 \\ 2 & -5 & 0 \end{bmatrix}$$

$$M_{23} = \begin{vmatrix} 1 & -2 \\ 2 & -5 \end{vmatrix} = 1(-5) - (-2)(2) = -5 + 4$$

$$M_{23} = -1$$

$$M_{12} = \begin{vmatrix} -3 & 1 \\ 2 & 0 \end{vmatrix} = -3(0) - 1(2) = 0 - 2$$

$$M_{12} = -2$$

## Cofactor of a matrix:

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$\begin{bmatrix} 3 & 1 & 5 \\ -4 & -2 & 1 \\ 2 & 1 & -3 \end{bmatrix}$$

$$C_{12} = (-1)^{1+2} M_{12}$$

row  $\rightarrow$  column  $= -1 \cdot$

$$\begin{vmatrix} -4 & 1 \\ 2 & -3 \end{vmatrix}$$

$$C_{33} = (-1)^{3+3} M_{33}$$

$$= 1 \cdot \begin{vmatrix} 3 & 1 \\ -4 & -2 \end{vmatrix}$$

$$= -1[-4(-3) - 1(2)]$$

$$= -1(12 - 2)$$

$$= 3(-2) - 1(-4)$$

$$= -6 + 4$$

$$C_{12} = -10$$

$$C_{33} = -2$$

**Determinant** of a 3 x 3 matrix:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot C_{11} + a_{12} \cdot C_{12} + a_{13} \cdot C_{13}$$

Find the value of the determinant:

$$\begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 0 \\ 3 & -1 & 2 \end{vmatrix} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$= 2 \cdot (-1)^{1+1} M_{11} - 1 \cdot (-1)^{1+2} M_{12} - 3 \cdot (-1)^{1+3} M_{13}$$

$$= 2M_{11} + M_{12} - 3M_{13}$$

$$= 2 \begin{vmatrix} 2 & 0 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= 2(4-0) + (2-0) - 3(-1-6)$$

$$= 8 + 2 + 21$$

$$D = 31$$

Find the determinant of each matrix.

$$\begin{vmatrix} 3 & -4 & -1 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{vmatrix} = -a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} \\
 = 3(-1)^{1+1}M_{11} - 4(-1)^{1+2}M_{12} - 1(-1)^{1+3}M_{13} \\
 = 3 \begin{vmatrix} -3 & 2 \\ 2 & -3 \end{vmatrix} + 4 \begin{vmatrix} 2 & 2 \\ 2 & -3 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 2 & 2 \end{vmatrix} \\
 = 3(9-4) + 4(-6-4) - 1(4+6) \\
 = 15 - 40 - 10 \\
 \textcircled{D} = -35$$

Assignment:

p.496 #1-11 odd