Today's Plan:

Learning Target (standard): I will solve real-world rates of change application problems.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make neccessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

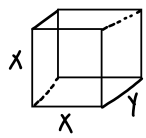
Teacher will: Provide practice problems over previous concepts, check homework problems for accuarcy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

A cereal box with a height that equals its length has a volume of 100 in^3 and is to be manufactured. What dimensions will minimize the cost of material?

SA



$V = x^2 y$

$100 = x^2 y \qquad y = \frac{100}{x^2}$

$SA = 2x^2 + 4xy$

$SA(x) = 2x^2 + 4x\left(\frac{100}{x^2}\right)$

$SA(x) = 2x^2 + 400x^{-1}$

$SA'(x) = 4x - 400x^{-2}$

$0 = 4x^{-2}(x^3 - 100)$

$0 = 4x^{-2}(x - \sqrt[3]{100})(x^2 + \sqrt[3]{100}x + \sqrt[3]{10000})$

Critical #s:

$x = \cancel{0}, \sqrt[3]{100}$

$SA''(x) = 4 + 800x^{-3}$

$SA''(\sqrt[3]{100}) = 4 + \frac{800}{(\sqrt[3]{100})^3} > 0$ minimum@ $x = \sqrt[3]{100}$

$y = \frac{100}{x^2} = \frac{100}{(\sqrt[3]{100})^2}$

$= \frac{100}{100^{2/3}}$

$= 100^{1/3}$

$y = \sqrt[3]{100}$

$\boxed{\begin{array}{l}\text{dimensions:}\\ x = \sqrt[3]{100}\text{ in}\\ y = \sqrt[3]{100}\text{ in}\end{array}}$

Rate of Change

- A derivative is a "rate of change" that can be used in a number of situations
- Velocity is the "rate of change" of distance with respect to time
- Acceleration is the "rate of change" of velocity with respect to time

A spherical balloon with radius $\boxed{r(t) = 4t^2 + 2t}, 0 \le t \sec \le 5$ is given. If the radius is measured in cm, find the rate of change of each of the following:

1. the radius at 3 seconds
2. the volume
3. the surface area

① $r(t) = 4t^2 + 2t$

$r'(t) = 8t + 2$

$r'(3) = 8(3) + 2$

$r'(3) = 26 \text{ cm/sec}$

$r'(t) = \frac{dr}{dt} = \frac{\text{cm}}{\text{sec}}$

② $V = \frac{4}{3}\pi r^3$

$V(t) = \frac{4}{3}\pi(4t^2+2t)^3$

$V'(t) = 4\pi(4t^2+2t)^2(8t+2)$

$= 4\pi(2t(2t+1))^2(2(4t+1))$

$= 4\pi \cdot 4t^2(2t+1)^2 \cdot 2(4t+1)$

$V'(t) = 32\pi t^2(2t+1)^2(4t+1) \text{ cm}^3/\text{sec}$

$V'(t) = \frac{dV}{dt} = \frac{\text{cm}^3}{\text{sec}}$

③ $SA = 4\pi r^2$

$SA(t) = 4\pi(4t^2+2t)^2$

$SA'(t) = 8\pi(4t^2+2t)(8t+2)$

$= 8\pi \cdot 2t(2t+1) \cdot 2(4t+1)$

$SA'(t) = 32\pi t(2t+1)(4t+1) \text{ cm}^2/\text{sec}$

$SA'(t) = \frac{dSA}{dt} = \frac{\text{cm}^2}{\text{sec}}$

Motion of a Particle

- When the velocity is negative the particle is moving to the left.
- When the velocity is positive the particle is moving to the right.
- When the velocity and acceleration of the particle have the same signs, the particle's speed is increasing.

* Speed is non-directional velocity

speed = $| v(t) |$

Motion continued

- When the velocity and acceleration of the particle have the opposite signs, the particle's speed is decreasing (or slowing down).
- When the velocity is zero and the acceleration is not zero, the particle is momentarily stopped and changing direction.

Assignment:

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