Today's Plan:

Learning Target (standard): I will estimate the area under a curve using Riemann sums.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Homework:

1. Estimate the area under $y = x^3$ on the interval [2,3] using:

a. 4 inscribed rectangles $A \approx \frac{893}{64}u^2$

b. 4 circumscribed rectangles $A \approx \frac{1197}{64}u^2$

c. 4 trapezoids $A \approx \frac{1045}{64}u^2$

d. 4 mid-points $A \approx \frac{2075}{128}u^2$

Homework:

2. Estimate the area under $y = 2x - x^2$ on the interval [1,2] using:

a. 4 inscribed rectangles $A \approx \frac{17}{32}u^2$

b. 4 circumscribed rectangles $A \approx \frac{25}{32}u^2$

c. 4 trapezoids $A \approx \frac{21}{32}u^2$

Describe the factors that play a role in estimating the area under a curve using Riemann sums.

- Number of subintervals
- Type of Riemann Sum
  - Left-Endpoint
    - concave down - over estimate
    - concave up - under estimate
  - Right-Endpoint
    - concave up - over estimate
    - concave down - under estimate

depending on the function

  - Inscribed - under estimate
  - Circumscribed - over estimate
  - Trapezoid - closest
  - Mid-Point - inscribed/circumscribed
- "Slope" of the function
- Interval length

Estimate the area under $y = 4 - x^2$ on the interval [-1,1] using:

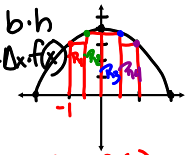
a. 4 inscribed rectangles
b. 4 circumscribed rectangles
c. 4 trapezoids

$\Delta x = \frac{b-a}{n}$ $[a,b]$

n ← # of intervals

$A = b \cdot h$

$A = \Delta x \cdot f(x)$



$\Delta x = \frac{1+1}{4} = \frac{1}{2}$

"width of a rectangle"

$A_{R_1} = \Delta x \cdot f(-1)$
$= \frac{1}{2}(3)$
$A_{R_1} = \frac{3}{2} u^2$

$A_{R_2} = \Delta x \cdot f(-\frac{1}{2})$
$= \frac{1}{2}(\frac{15}{4})$
$A_{R_2} = \frac{15}{8} u^2$

$A_{R_3} = \Delta x \cdot f(\frac{1}{2})$
$= \frac{1}{2}(\frac{15}{4})$
$A_{R_3} = \frac{15}{8} u^2$

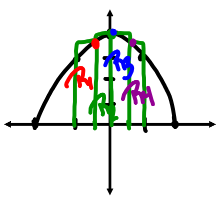
$A_{R_4} = \Delta x \cdot f(1)$
$= \frac{1}{2}(3)$
$A_{R_4} = \frac{3}{2} u^2$

$A \approx A_{R_1} + A_{R_2} + A_{R_3} + A_{R_4}$
$\approx \frac{3}{2} + \frac{15}{8} + \frac{15}{8} + \frac{3}{2}$
$A \approx \frac{54}{8}$
$A \approx \frac{27}{4} u^2$

Estimate the area under $y = 4 - x^2$ on the interval [-1,1] using:

b. 4 circumscribed rectangles
c. 4 trapezoids

$\Delta x = \frac{1}{2}$



$A_{R_1} = \Delta x \cdot f(-\frac{1}{2})$
$= \frac{1}{2}(\frac{15}{4})$
$A_{R_1} = \frac{15}{8} u^2$

$A_{R_2} = \Delta x \cdot f(0)$
$= \frac{1}{2}(4)$
$A_{R_2} = 2 u^2$

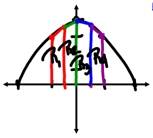
$A_{R_3} = \Delta x \cdot f(0)$
$= \frac{1}{2}(4)$
$A_{R_3} = 2 u^2$

$A_{R_4} = \Delta x \cdot f(\frac{1}{2})$
$= \frac{1}{2}(\frac{15}{4})$
$A_{R_4} = \frac{15}{8} u^2$

$A \approx A_{R_1} + A_{R_2} + A_{R_3} + A_{R_4}$
$\approx \frac{15}{8} + 2 + 2 + \frac{15}{8}$
$A \approx \frac{31}{4} u^2$

Estimate the area under $y = 4 - x^2$ on the interval [-1,1] using:

c. 4 trapezoids



$\Delta x = \frac{1}{2}$

$h = \frac{1}{2}$

$A_{trap} = \frac{1}{2}(b_1+b_2)h$

$A_{R_1} = \frac{1}{2}\left(f(-1)+f(-\frac{1}{2})\right)\cdot\frac{1}{2}$

$= \frac{1}{4}\left(3+\frac{15}{4}\right)$

$= \frac{1}{4}\left(\frac{27}{4}\right)$

$A_{R_1} = \frac{27}{16}u^2$

$A_{R_2} = \frac{1}{2}\left(f(-\frac{1}{2})+f(0)\right)\cdot\frac{1}{2}$

$= \frac{1}{4}\left(\frac{15}{4}+4\right)$

$= \frac{1}{4}\left(\frac{31}{4}\right)$

$A_{R_2} = \frac{31}{16}u^2$

$A_{R_3} = \frac{1}{2}\left(f(0)+f(\frac{1}{2})\right)\cdot\frac{1}{2}$

$= \frac{1}{4}\left(4+\frac{15}{4}\right)$

$= \frac{1}{4}\left(\frac{31}{4}\right)$

$A_{R_3} = \frac{31}{16}u^2$

$A_{R_4} = \frac{1}{2}\left(f(\frac{1}{2})+f(1)\right)\cdot\frac{1}{2}$

$= \frac{1}{4}\left(\frac{15}{4}+3\right)$

$= \frac{1}{4}\left(\frac{27}{4}\right)$

$A_{R_4} = \frac{27}{16}u^2$

$A \approx A_{R_1}+A_{R_2}+A_{R_3}+A_{R_4}$

$\approx \frac{27}{16}+\frac{31}{16}+\frac{31}{16}+\frac{27}{16}$

$A \approx \frac{29}{4}u^2$

For each problem, approximate the area under the curve over the given interval using 4 left endpoint rectangles.

1) $y = -x^2 - 2x + 10$; $[-4, 0]$ $A_{R_1} = 2u^2$ $A_{R_2} = 7u^2$ $A_{R_3} = 10u^2$ $A_{R_4} = 11u^2$

$A \approx 30u^2$

For each problem, approximate the area under the curve over the given interval using 4 midpoint rectangles.

2) $y = \frac{x}{2} + 6$; $[-3, 5]$

$A \approx 52u^2$

For each problem, approximate the area under the curve over the given interval using 4 circumscribed rectangles.

3) $y = -x + 4$; $[-5, 3]$

$A \approx 48u^2$

For each problem, approximate the area under the curve over the given interval using 4 right endpoint rectangles.

4) $y = -\frac{x^2}{2} + 6$; $[-1, 3]$

$A \approx 17u^2$

For each problem, approximate the area under the curve over the given interval using 4 inscribed rectangles.

5) $y = -\frac{x}{2} + 3$; $[-3, 5]$

$A \approx 16u^2$

For each problem, approximate the area under the curve over the given interval using 4 trapezoids.

6) $y = x^2 + 2x + 3$; $[-4, 0]$

$A \approx 18u^2$