

Today's Plan:

Learning Target (standard): I will use the sum and difference trigonometric identities to evaluate expressions.

Students will: Complete practice problems over previous concepts at the boards, put up homework problems on the board and make necessary corrections to their own work, take notes over new material and complete practice problems over new concepts.

Teacher will: Provide practice problems over previous concepts, check homework problems for accuracy and provide students feedback, describe and provide examples of new concepts and assign students assessment problems over new concepts.

Assessment: Board work, homework check and homework assignment

Differentiation: Students will work at the board, go over and correct homework at their seats, actively engage in lecture over new concepts, practice new concepts with the aid of other students and the teacher and complete homework assignment.

Take a few minutes to compare last night's identities before we go on to the next set.



$$\begin{aligned}
 & 74) (\tan \alpha + \tan \beta)(1 - \cot \alpha \cot \beta) + (\cot \alpha + \cot \beta) \\
 & \quad \quad \quad (1 - \tan \alpha \tan \beta) \\
 & \quad \quad \quad = 0 \\
 & \tan \alpha - \tan \alpha \cot \alpha \cot \beta + \tan \beta - \tan \beta \cot \alpha \cot \beta \\
 & \quad + \cot \alpha - \cot \alpha \tan \alpha \tan \beta + \cot \beta - \cot \beta \tan \alpha \tan \beta \\
 & \tan \alpha - \cancel{\tan \alpha} \left(\frac{1}{\tan \alpha} \right) \cot \beta + \tan \beta - \cancel{\tan \beta} \cot \alpha \left(\frac{1}{\tan \beta} \right) \\
 & \quad + \cot \alpha - \left(\frac{1}{\tan \alpha} \right) \tan \alpha \tan \beta + \cot \beta \\
 & \quad \quad \quad - \left(\frac{1}{\tan \beta} \right) \tan \alpha \tan \beta \\
 & \cancel{\tan \alpha} - \cancel{\cot \beta} + \cancel{\tan \beta} - \cancel{\cot \alpha} + \cancel{\cot \alpha} - \cancel{\tan \beta} \\
 & \quad \quad \quad + \cancel{\cot \beta} - \cancel{\tan \alpha} \\
 & 0 \therefore \text{Q.E.D.}
 \end{aligned}$$

Establish the identity.

$$\ln |1 + \cos \theta| + \ln |1 - \cos \theta| = 2 \ln |\sin \theta|$$

$$\ln |(1 + \cos \theta)(1 - \cos \theta)|$$

$$\begin{aligned}
 \log_a x + \log_a y &= \\
 \log_a (xy)
 \end{aligned}$$

$$\ln |1 - \cancel{\cos \theta} + \cancel{\cos \theta} - \cos^2 \theta|$$

$$\ln |1 - \cos^2 \theta|$$

$$\ln |(\sin^2 \theta + \cos^2 \theta) - \cos^2 \theta|$$

$$\ln |\sin^2 \theta|$$

$$2 \ln |\sin \theta| \therefore \text{Q.E.D.}$$

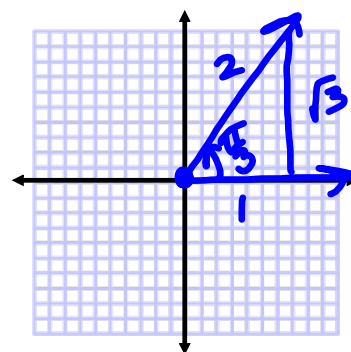
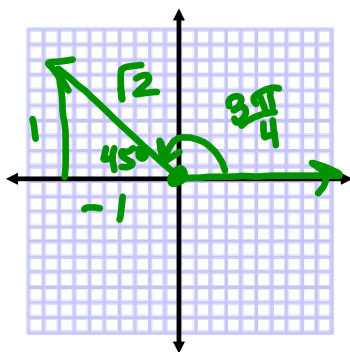
Evaluate.

$$\sin \frac{3\pi}{4} - \csc \frac{\pi}{3}$$

$$= \frac{\sqrt{2}}{2} - \frac{2\sqrt{3}}{3}$$

$$= \frac{3\sqrt{2}}{6} - \frac{4\sqrt{3}}{6}$$

$$= \frac{3\sqrt{2} - 4\sqrt{3}}{6}$$

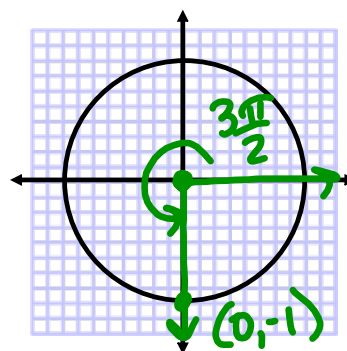
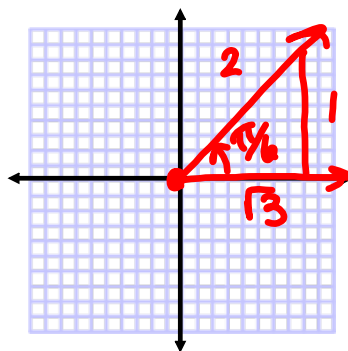


Evaluate.

$$\tan^2 \frac{\pi}{6} + \cos \frac{3\pi}{2}$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 + 0$$

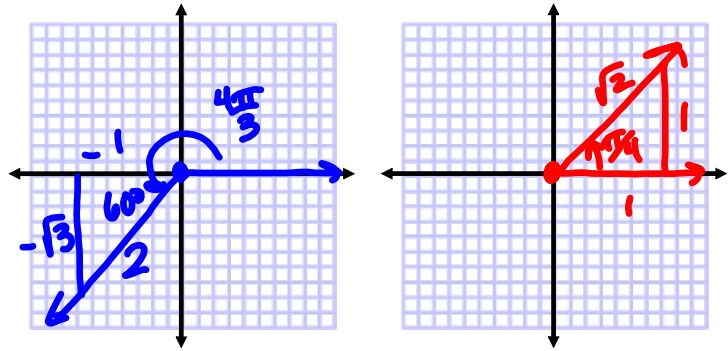
$$= \frac{1}{3}$$



$$\begin{aligned} x &= 0 \\ y &= -1 \\ r &= 1 \end{aligned}$$

Evaluate.

$$\begin{aligned}
 & 2\sec^2 \frac{4\pi}{3} - \cot \frac{\pi}{4} \\
 &= 2(-2)^2 - 1 \\
 &= 8 - 1 \\
 &= 7
 \end{aligned}$$



Sum and Difference Identities: α and β are common angles!

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

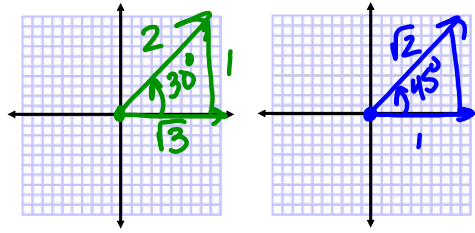
$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \quad \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

* These are used to find the exact value of trigonometric expressions containing angles that are NOT common

Find the exact value.

$$\sin \frac{5\pi}{12}$$



$$= \sin 75^\circ$$

$$= \sin(\overset{\alpha}{30^\circ} + \overset{\beta}{45^\circ})$$

$$= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$$

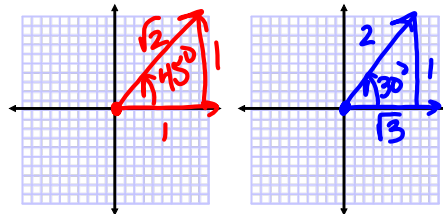
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

Find the exact value.

$$\tan 15^\circ$$



$$= \tan(\overset{\alpha}{45^\circ} - \overset{\beta}{30^\circ})$$

$$= \tan 45^\circ - \tan 30^\circ$$

$$\frac{1 + \tan 45^\circ \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$

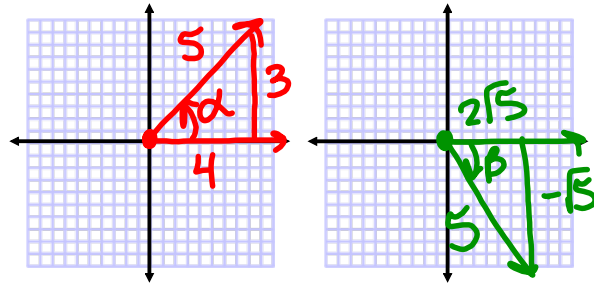
$$= \frac{9 - 3\sqrt{3} - 3\sqrt{3} + 3}{9 - 3} = \frac{12 - 6\sqrt{3}}{6}$$

$$= 2 - \sqrt{3}$$

Find the exact value.

$$\sin \alpha = \frac{3}{5}, 0 < \alpha < \frac{\pi}{2}$$

$$\cos \beta = \frac{2\sqrt{5}}{5}, -\frac{\pi}{2} < \beta < 0$$

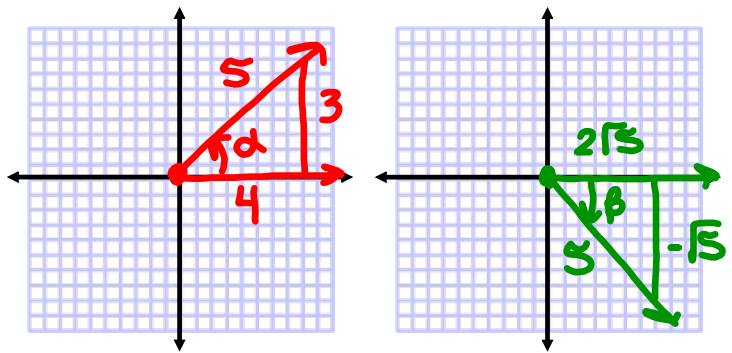


$$\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \left(\frac{4}{5}\right)\left(\frac{2\sqrt{5}}{5}\right) - \left(\frac{3}{5}\right)\left(-\frac{\sqrt{5}}{5}\right) \\ &= \frac{8\sqrt{5}}{25} + \frac{3\sqrt{5}}{25} \\ &= \frac{11\sqrt{5}}{25} \end{aligned}$$

Find the exact value.

$$\sin \alpha = \frac{3}{5}, 0 < \alpha < \frac{\pi}{2}$$

$$\cos \beta = \frac{2\sqrt{5}}{5}, -\frac{\pi}{2} < \beta < 0$$



$$\sin(\alpha + \beta) =$$

Assignment:

p.470 #4-28 (by 4)

* Write the problem, draw appropriate diagrams, write the formula & show ALL work *